A certain genetic testing company takes between 1 and 9 weeks to analyze a patient's DNA profile. **SCORE: _____ / 7 PTS**

Let X be the random variable representing the time (in weeks) taken to process a randomly selected profile. Find the mean (average) processing time if the probability density function is given by

$$f(x) = \begin{cases} kx^{-\frac{3}{2}}, & x \in [1, 9] \\ 0, & x \notin [1, 9] \end{cases}$$ (for some appropriate constant k).

$\boxed{\int_1^9 kx^{-\frac{3}{2}}\,dx = 1}$ 1

1 $\boxed{-2kx^{-\frac{1}{2}}}\Big|_1^9 = 1$

$-2k\left(\frac{1}{3} - 1\right) = 1$

$\boxed{k = \frac{3}{4}}$ 1

$\overline{X} = \int_1^9 x \cdot \frac{3}{4} x^{-\frac{3}{2}}\,dx$

$= \boxed{\frac{3}{4}\int_1^9 x^{-\frac{1}{2}}\,dx}$ $1\frac{1}{2}$

$= \frac{3}{4} \cdot \boxed{2x^{\frac{1}{2}}}\Big|_1^9$ 1

$= \frac{3}{2}(3-1)$

$= \boxed{3}$ $\boxed{\text{WEEKS}}$ $\frac{1}{2}$

1

For the function $f(x) = x^2 - 2x$ on the interval $x \in [-4, 2]$, find the value of c such that $f_{ave} = f(c)$. **SCORE: _____ / 7 PTS**

NOTE: This is the value c guaranteed by the Mean Value Theorem for Integrals.

$$f_{AVE} = \boxed{\frac{1}{2 - -4} \int_{-4}^{2} (x^2 - 2x)\,dx} \quad 2$$

$$= \frac{1}{6} \boxed{\left(\frac{1}{3}x^3 - x^2\right)} \Big|_{-4}^{2}$$

1

$$= \frac{1}{6}\left(\frac{1}{3}(8 - -64) - (4 - 16)\right)$$

$$= \boxed{6} \quad 1$$

$$f(c) = f_{AVE}$$

$$\boxed{c^2 - 2c = 6} \quad 1$$

$$c^2 - 2c - 6 = 0$$

$$c = \frac{2 \pm \sqrt{4 + 24}}{2} = \frac{2 \pm 2\sqrt{7}}{2}$$

$$= \boxed{1 \pm \sqrt{7}} \quad 1$$

$$\boxed{c = 1 - \sqrt{7}} \in [-4, 2]$$

1

Find the length of the curve $y = \frac{1}{4}e^{3x} + \frac{1}{9}e^{-3x}$ on the interval $x \in [-2, 2]$. **SCORE: _____ / 8 PTS**

$$\boxed{\int_{-2}^{2} \sqrt{1 + \left(\frac{3}{4}e^{3x} - \frac{1}{3}e^{-3x}\right)^2}\, dx} \quad 3$$

$$= \int_{-2}^{2} \sqrt{1 + \left(\boxed{\frac{9}{16}e^{6x} - \frac{1}{2} + \frac{1}{9}e^{-6x}}\right)}\, dx \quad 1$$

$$= \int_{-2}^{2} \sqrt{\frac{9}{16}e^{6x} + \frac{1}{2} + \frac{1}{9}e^{-6x}}\, dx$$

$$= \int_{-2}^{2} \left(\boxed{\frac{3}{4}e^{3x} + \frac{1}{3}e^{-3x}}\right) dx \quad 1$$

$$= \left(\boxed{\frac{1}{4}e^{3x} - \frac{1}{9}e^{-3x}}\right)\Big|_{-2}^{2} \quad 1$$

$$= \boxed{\frac{1}{4}\left(e^{6} - \frac{1}{e^{6}}\right) - \frac{1}{9}\left(\frac{1}{e^{6}} - e^{6}\right)} \quad 1$$

$$= \boxed{\frac{13}{36}\left(e^{6} - \frac{1}{e^{6}}\right)} \quad 1$$

$$= \frac{13(e^{12} - 1)}{36e^{6}}$$

Find the surface area if the curve $y = \frac{x^5}{20} + \frac{1}{3x^3}$ for $x \in [1, 2]$ is revolved around the $y-$ axis. **SCORE: ______ / 8 PTS**

$$\int_1^2 \boxed{2\pi x} \boxed{\sqrt{1+\left(\frac{1}{4}x^4 - x^{-4}\right)^2}\, dx} \ 2$$

1

$$= 2\pi \int_1^2 x\sqrt{1+\boxed{\left(\frac{1}{16}x^8 - \frac{1}{2} + x^{-8}\right)}}\, dx$$

1

$$= 2\pi \int_1^2 x\sqrt{\frac{1}{16}x^8 + \frac{1}{2} + x^{-8}}\, dx$$

$$= 2\pi \int_1^2 x\left(\boxed{\frac{1}{4}x^4 + x^{-4}}\right) dx \ 1$$

$$= 2\pi \int_1^2 \left(\boxed{\frac{1}{4}x^5 + x^{-3}}\right) dx \ 1$$

$$= 2\pi \left(\boxed{\frac{1}{24}x^6 - \frac{1}{2}x^{-2}}\right)\Big|_1^2 \ 1$$

$$= 2\pi \left(\frac{1}{24}(64-1) - \frac{1}{2}\left(\frac{1}{4} - 1\right)\right)$$

$$= 2\pi \left(\frac{\cancel{63}^{21}}{\cancel{24}_{8}} + \frac{3}{8}\right) = \boxed{6\pi} \ 1$$